Christ Church
Grammar School

2014

UNIT TEST 2

MATHEMATICS 3C/3D

Section One:
Calculator-free

Your name _____

Time allowed for this section

Working time for this section: fifteen minutes

Materials required/recommended for this section

To be provided by the supervisor

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Section One: Calculator-free **(15 marks)**

Answer all questions.

Working time: 15 minutes.

Question 1 **(8 marks)**

The events A and B have the properties $P(A) = \frac{3}{8}$ and $P(A \cup B) = \frac{1}{2}$.

(a) Determine $P(B)$ in each of the following cases:

(i) If A and B are mutually exclusive. (1 mark)

(ii) If $P(A \cap B) = \frac{3}{40}$. (2 marks)

Question 1 continued.

(iii) If $P(B \mid A) = \frac{1}{6}$. (2 marks)

(b) For the case where $P(A \cap B) = \frac{3}{40}$, are A and B independent? Justify your answer.

(2 marks)

Question 2 **(8 marks)**

Evaluate the following integrals

(a) $\int_1^4 3\sqrt{x}\ dx$ (3 marks)

(b) $\int (6x + 9)(3x + x^2)^2\ dx$ (2 marks)

(c) $\int x(3x^2 + 6x)^4 + (3x^2 + 6x)^4\ dx$ (3 marks)

End of questions

Christ Church Grammar School

UNIT TEST 2

MATHEMATICS 3C/3D

Section Two: Calculator-assumed

Your name _____

Time allowed for this section

Working time for this section: thirty minutes

Materials required/recommended for this section

To be provided by the supervisor

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: up to three calculators approved for use in the WACE examinations

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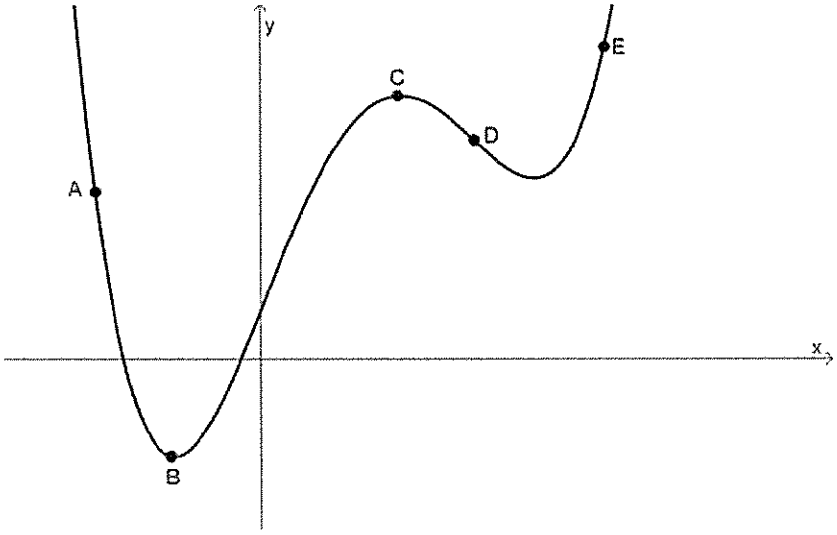
Section Two: Calculator-assumed **(30 marks)**

Answer all questions.

Working time: 30 minutes.

Question 3 **(5 marks)**



The diagram above shows the graph of $y = f(x)$.

Using one or more of the five given points A, B, C, D and E on the graph, list the point(s) on the graph of $y = f(x)$ that satisfy the given conditions.

(a) $f'(x) > 0$ and $f''(x) > 0$ Answer: _____ (1 mark)

(b) $f'(x) < 0$ and $f''(x) > 0$ Answer: _____ (1 mark)

(c) $f'(x) = 0$ and $f''(x) < 0$ Answer: _____ (1 mark)

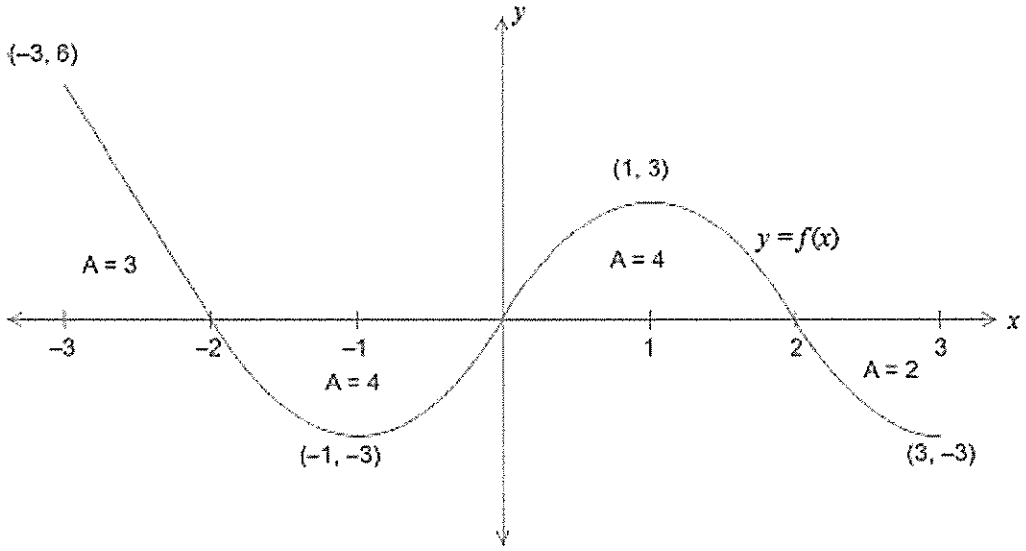
(d) $f'(x) = 0$ and $f''(x) > 0$ Answer: _____ (1 mark)

(e) $f'(x) < 0$ and $f''(x) = 0$ Answer: _____ (1 mark)

Question 4 **(9 marks)**

The graph of the function $f(x)$ is shown below for $-3 \leq x \leq 3$.

The areas enclosed between the graph, the x-axis and the lines $x = -3$ and $x = 3$ are marked in the appropriate regions in the diagram.



Determine

(a) the value of $\int_{-2}^{3} f(x)\,dx$ (2 marks)

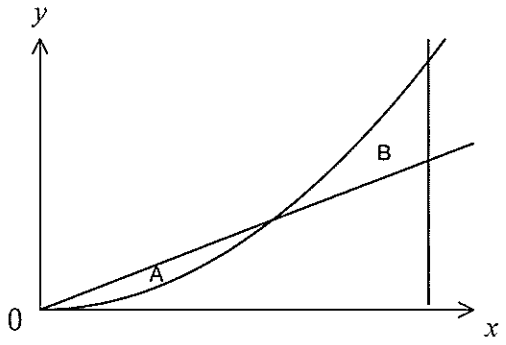
(b) the area enclosed between the graph of $f(x)$ and the x-axis, from $x = -2$ to $x = 3$. (2 marks)

(c) the value of $\int_{0}^{3} f(-x)\,dx$ (2 marks)

(d) the value of $\int_{0}^{2} x - f(x)\,dx$ (3 marks)

Question 5 **(3 marks)**

The diagram below, not to scale, shows the functions $f(x) = \frac{1}{10}x$, $g(x) = \frac{1}{10}x^2$ and the line $x = 2$.



Region A is the area trapped by the functions f and g.

Region B is the area trapped by the functions f, g and the line $x = 2$.

Find the area of region A and the area of region B? (3 marks)

Question 6 **(7 marks)**

(a) A team of 3 students is chosen at random from a group of 4 girls and 5 boys for a TV game show. What is the probability that the team chosen consists of more boys than girls?

(2 marks)

(b) In one of the games, the team choose one of four closed doors. The doors then open to reveal a prize placed at random behind just one of them. The team keep the prize if they are correct.

How many rounds of this game must the team play so that the probability of them obtaining at least one prize is greater than 0.95? (2 marks)

Question 6 continued.

(c) At the close of the show, the team can select **one** of two boxes, Box A or Box B, to keep as another prize. Inside each of the boxes are five sealed envelopes, each containing a voucher. In Box A, four of the vouchers are worth $10 000 and the fifth $100, whilst in Box B, two of the vouchers are worth $10 000 and the other three, $100 each.

The team is allowed to choose an envelope from one of the boxes and open it. They must then decide whether to keep that box or choose the other box. The team plan to keep the box that the envelope they opened came from if it contains a $10 000 voucher. Otherwise they will take the **other** box.

With the help of a probability tree or otherwise, find the probability that the team win more than $30 000?

Hints: There are **2 possible pathways** for the team to win more than $30 000 – it could come from drawing a $10 000 voucher from box A in the first instance or from drawing a $100 voucher from box B in the first instance.

(3 marks)

Question 7 **(6 marks)**

At the end of a technology course, all students sat a practical and a theory examination, with 20% achieving a distinction in the practical examination, 3% of students achieving distinctions in both examinations and 76% achieving no distinction in either examination.

(a) With the help of a Venn diagram, find the probability that a student chosen at random from the course achieved a distinction in the theory examination? (2 marks)

(b) Are the events 'achieving a distinction in the practical examination' and 'achieving a distinction in the theory examination' independent? Explain your answer. (2 marks)

(c) In a group of 14 students who took the course, three achieved a distinction in the practical examination. If five students are selected at random from this group, what is the probability that at least two of them achieved a distinction in the practical examination? (2 marks)

End of questions